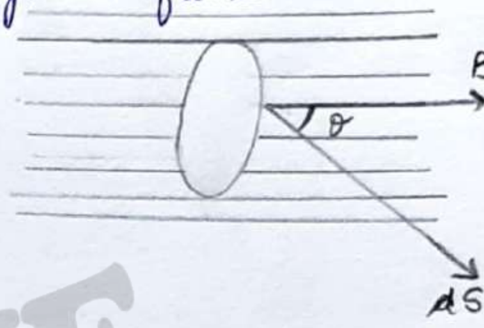
ELECTROMAGNETIC INDUCTION

MAGNETIC FLUX -

It represents total magnetic lines of force passing normally through a given area placed in a magnetic field.

$$\boxed{\phi_B = B \cdot S = BS\cos\theta}$$



Unit - Weber (Wb) = Tm^2 → SI unit
Maxwell → CGS unit

ELECTROMAGNETIC INDUCTION -

The phenomenon to generate induced current or induced emf by changing the magnetic flux linked with a closed circuit is known as Electromagnetic Induction.

FARADAY'S LAWS

① First Law - Whenever there is change in magnetic flux linked with the closed loop, an emf induces in the loop which lasts as long as the change in flux continues.

② Second Law - The induced emf in a closed loop or circuit is directly proportional to the rate of change of magnetic flux linked with the closed loop or circuit.

i.e. $e \propto (-)\dfrac{d\phi}{dt}$

$$\boxed{e = -\left(\frac{d\phi}{dt}\right)}$$

* The negative sign is due to Lenz Law.

LENZ LAW -

Current induced in the loop due to changing magnetic flux is such that it tends to oppose the rate of change of magnetic flux.

- Lenz law is in accordance with law of conservation of energy.

INDUCED CURRENT -

If N is the number of turns and R is the resistance of a coil, the magnetic flux linked with its each turn changes by $d\phi$ in short time interval dt, then induced current flowing through the coil is

$$e = -\frac{d\phi}{dt} \qquad I = \frac{|e|}{R} \quad \Rightarrow \quad I = -\frac{1}{R}\left(\frac{d\phi}{dt}\right) \text{ --- } (1)$$

$$\because I = \frac{dq}{dt}$$

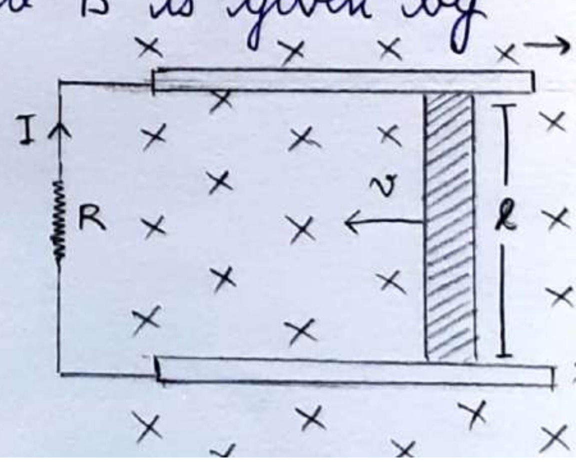
$$\frac{dq}{dt} = -\frac{1}{R}\left(\frac{d\phi}{dt}\right) \quad \text{using } (1)$$

$$\boxed{q = -\frac{1}{R}\int d\phi}$$

MOTIONAL EMF -

The potential difference induced in a conductor of length l moving with ~~vo~~ velocity v in a direction perpendicular to magnetic field B is given by

$$\mathcal{E} = \int (v \times B)\cdot dl = vBl$$



$B \rightarrow$ magnetic field
$l \rightarrow$ length of conducting wire
$v \rightarrow$ velocity

$$e = -\frac{d\phi}{dt}$$

$d\phi = BA$ $\qquad$ $e = -\frac{d}{dt}(BA)$

$A = lx$ $\qquad$ $e = -\frac{d}{dt} B(lx)$

$$\boxed{e = Blv}$$

• emf will not be induced if any two of Blv are parallel.

FORCE

$$I = \frac{Blv}{R}$$

$$F = BIl$$

$$\boxed{F = \frac{B^2l^2v}{R}}$$

POWER

$$P = F \times v$$

$$\boxed{P = \frac{B^2l^2v^2}{R}}$$

EDDY CURRENT

The current induced in bulk piece of conductor when magnetic flux linked with the conductor changes is known as eddy currents.

$$\boxed{i = \frac{e}{R}}$$

Applications -

1. Magnetic Braking
2. Induction furnace
3. Speedometer
4. Electromagnetic damping
5. Energy meter

Disadvantages -

1. Lot of heat energy is produced which damages the core of material.
2. Excessive heating may lead to fire.
3. Reduces the efficiency of the machine.

Ways to minimize -

1. By laminating the core.
2. By making slots on the conductor surface.

INDUCTANCE -

The flux linkage of a closely wound coil is directly proportional to the current I i.e. $\phi_B \propto I$. The flux linked with the coil having 'N' turns will be $N\phi_B \propto I$. The constant of proportionality in this relation is called inductance.

SELF INDUCTANCE

The phenomenon of production of induced emf in a coil, when a current pass through it, undergoes a change.

$\because$ Total flux linked with coil, $N\phi \propto I$

$$\boxed{N\phi = LI}$$

where, ϕ = flux linked with each turn and
L = coefficient of self - induction or self inductance

Also, induced emf, $e = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$

SI unit - Henry (H)

where, $L = \frac{\varepsilon}{\quad}$

Self Inductance of Long Solenoid -

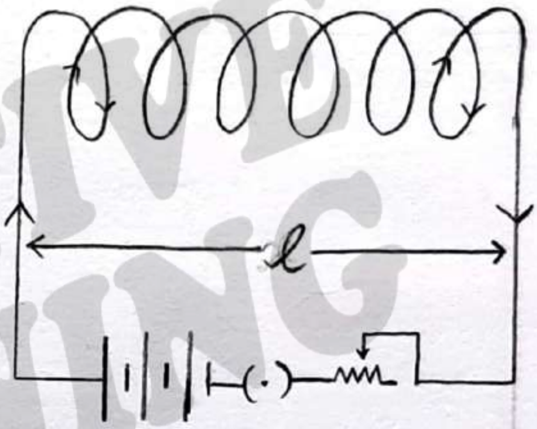
The magnetic field B at any point inside such a solenoid is constant;

$$B = \frac{\mu_0 N I}{l} = \mu_0 n I$$

where,
μ_0 = magnetic permeability
N = total no. of turns
l = length of the solenoid
n = no. of turns per unit length

ϕ = B × area of each turn

$$\phi = \left(\mu_0 \frac{N}{l} I\right) A$$



$$N\phi = L I$$

where $L = \frac{\mu_0 N^2 A}{l}$

$$\boxed{L = \frac{\mu_0 \mu_r N^2 A}{l}}$$

MUTUAL INDUCTANCE -

The phenomenon according to which an opposing emf is produced in a coil as a result of change in current or magnetic flux linked with a neighbouring coil is called inductance.

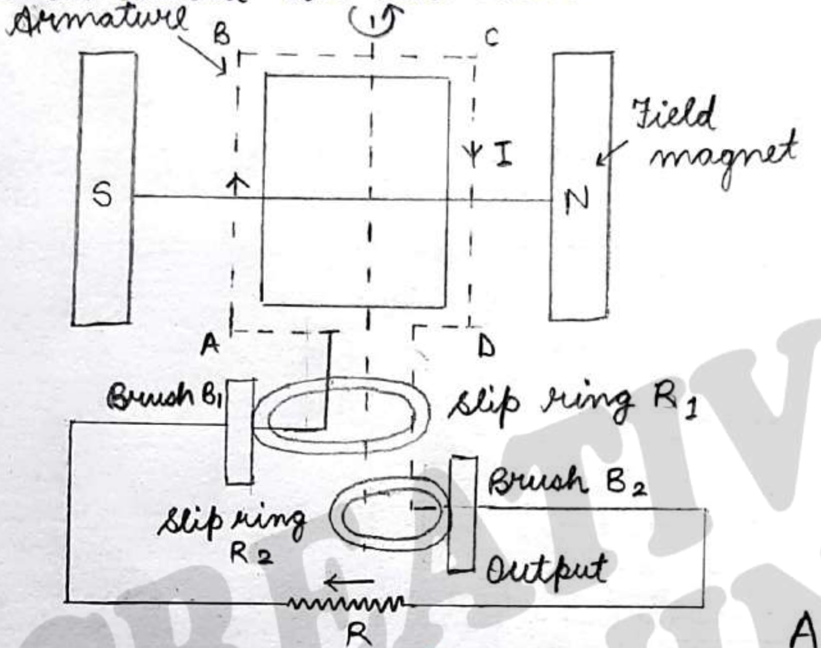
$$\phi \propto I$$

$$\phi = M I$$

$$\boxed{e = -M \frac{dI}{dt}}$$

AC GENERATOR

An AC generator is based on the phenomenon of electromagnetic induction, which states that a coil is rotated in uniform magnetic field, the magnetic flux linked with a conductor changes and an emf is induced in the coil.



AC GENERATOR

Theory and Working

As the armature of coil is rotated in uniform magnetic field, angle θ changes continuously. Therefore, magnetic flux changes and an emf is induced. If e is the emf induced in the coil, then

$$e = -\frac{N d\phi}{dt} \quad \text{or} \quad e = -\frac{d}{dt}(NBA \cos \omega t)$$

$$\varepsilon = NBA\omega \sin \omega t$$

N → no. of turns in the coil
B → strength of magnetic field
A → area of each turn of coil
ω → angular velocity of rotation of the coil

and $I = \frac{e}{R} = \frac{NBA\omega}{R} \sin \omega t$, R → resistance of the coil

IMPORTANT QUESTIONS

1. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside, normal to the axis of solenoid. If the current carried by the solenoid changes steadily from 2 A to 4 A in 0.1 s, what is the induced voltage in the loop, while the current is changing? (NCERT)

Sol. Here, no. of turns per unit length,

$$n = \frac{N}{l} = 15 \text{ turns/cm} = 1500 \text{ turns/m}$$

$$A = \frac{N}{l} = \cancel{15\ tur}\ 2.0\ cm^2 = 2 \times 10^{-4}\ m^2$$

$$\frac{dI}{dt} = \frac{4-2}{0.1} \text{ or } \frac{dI}{dt} = 20\ A s^{-1}$$

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA)$$

$$|e| = \frac{A\,d}{dt}\left(\mu_0 \frac{NI}{l}\right) = A\mu_0\left(\frac{N}{l}\right)\frac{dI}{dt}$$

$$|e| = (2 \times 10^{-4}) \times 4\pi \times 10^{-7} \times 1500 \times 20\ V$$

$$|e| = 7.5 \times 10^{-6}\ V$$

2. A 1 m long conducting rod rotates with an angular frequency of 400 rad/s about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring. (NCERT)

Sol. Length of rod, $l = 1$ m, $\omega = 400$ rad s^{-1}, $B = 0.5$ T, $e = ?$

Average linear velocity,

$$v = \frac{0 + l\omega}{2} = \frac{l\omega}{2},\quad e = Blv$$

$$e = Bl\frac{l\omega}{2} = \frac{Bl^2\omega}{2} = \frac{0.5 \times (1)^2 \times 400}{2} = 100V$$

3. A line charge λ per ~~p~~ unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis as shown in the figure.

Sol. Given, linear charge density, $\lambda = \frac{\text{Total charge}}{\text{length}} = \frac{Q}{2\pi R}$

where, radius of rim = R and mass of rim = M

Magnetic field extends over a circular region,

$$B = -B_0 \hat{k} \, (r \leq a, a < R) = 0$$

Let the angular velocity of the wheel be ω, then the induced emf, $e = -\frac{d\phi}{dt}$

$$e = -\int E . dl = -\frac{d\phi}{dt}$$

$$E \int dl = -\frac{d}{dt}(\pi a^2 B)$$

$$E \times 2\pi a = -\pi a^2 \frac{dB}{dt} \; ; \; E = -\frac{a}{2} \cdot \frac{dB}{dt}$$

Force on charge, $F = QE = -\pi a^2 \lambda \frac{dB}{dt}$

$$F = \frac{dp}{dt} = M \cdot \frac{dv}{dt}$$

$$M \frac{dv}{dt} = -\pi a^2 \lambda \frac{dB}{dt} \Rightarrow MR\left(\frac{d\omega}{dt}\right) = -\pi a^2 \lambda \frac{dB}{dt}$$

$$d\omega = -\frac{\pi a^2 \lambda}{MR} dB$$

$$\omega = -\frac{\pi a^2 \lambda B}{MR} \Rightarrow \boxed{\omega = -\frac{\lambda a^2 \pi}{MR} B \hat{k}}$$